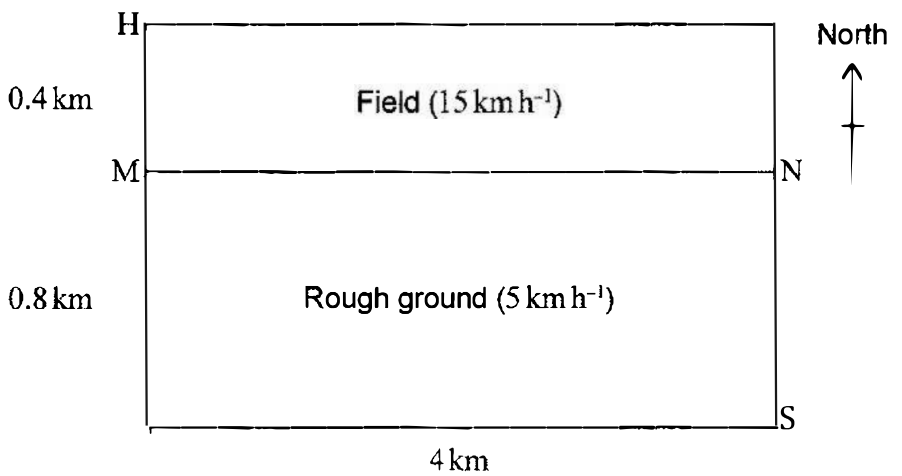
Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

This question considers the optimal route between two points, separated by several regions where different speeds are possible.

Huw lives in a house, H, and he attends a school, S, where H and S are marked on the following diagram. The school is situated $1.2\,\text{km}$ south and $4\,\text{km}$ east of Huw's house. There is a boundary [MN], going from west to east, $0.4\,\text{km}$ south of his house. The land north of [MN] is a field over which Huw runs at 15 kilometres per hour ($\text{km}\,\text{h}^{-1}$). The land south of [MN] is rough ground over which Huw walks at $5\,\text{km}\,\text{h}^{-1}$. The two regions are shown in the following diagram.

diagram not to scale



(a) Huw travels in a straight line from H to S. Calculate the time that Huw takes to complete this journey. Give your answer correct to the nearest minute. [6]

(b) Huw realizes that his journey time could be reduced by taking a less direct route. He therefore defines a point P on [MN] that is $x\,\text{km}$ east of M. Huw decides to run from H to P and then walk from P to S Let $T(x)$ represent the time, in hours, taken by Huw to complete the journey along this route.

(i) Show that $T(x)$ $\dfrac{\sqrt{0.4^2+x^2}+3\sqrt{0.8^2+(4-x)^2}}{15}$ [3]

(ii) Sketch the graph of $y=T(x)$. [2]

(This question continues on the following page)

(Question 1 continued)

(iii) Hence determine the value of x that minimizes $T(x)$. [1]

(iv) Find by how much Huw's journey time is reduced when he takes this optimal route, compared to travelling in a straight line from H to S. Give your answer correct to the nearest minute. [2]

(c) (i) Determine an expression for the derivative $T'(x)$. [3]

(ii) Hence show that $T(x)$ is minimized when

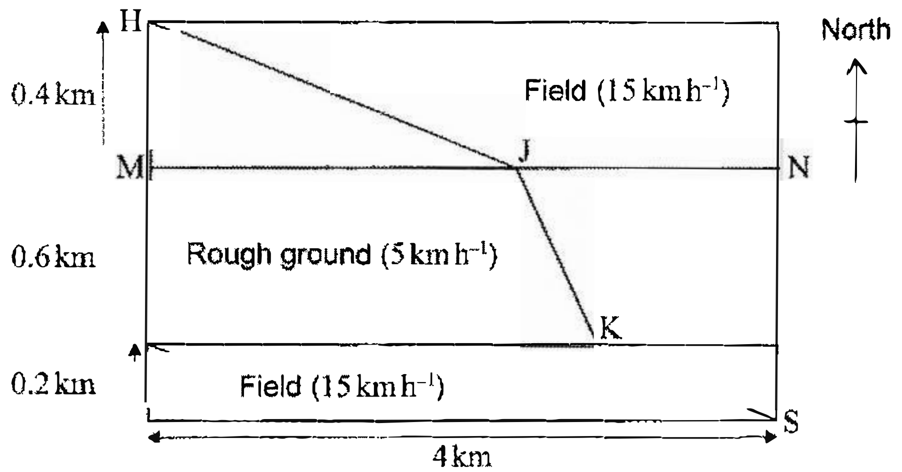
$$\frac{x}{\sqrt{0.16+x^2}} = \frac{3(4-x)}{\sqrt{0.64+(4-x)^2}}.$$ [1]

(iii) For the optimal route, verify that the equation in part (c)(ii) satisfies the following result:

$$\frac{\cos \mathrm{H\hat{P}M}}{\cos \mathrm{S\hat{P}N}} = \frac{\text{speed over field}}{\text{speed over rough ground}}$$ [2]

(d) The owner of the rough ground converts the southern quarter into a field over which Huw can run at $15\,\mathrm{km\,h^{-1}}$. The following diagram shows the optimal route, HJKS, in this new situation. You are given that [HJ] is parallel to [KS].

diagram not to scale



Using a similar result to that given in part (c)(iii), at the point J, determine MJ. [6]

2. [Maximum mark: 29]

This question considers the analysis of several datasets of examination marks using a variety of standard procedures and also an unfamiliar statistical test.

A class of eight students sits two examinations, one in French and one in German.

The marks in these examinations are given in **Table 1**.

Table 1

Student	French mark	German mark
S_1	42	39
S_2	65	66
S_3	82	71
S_4	50	53
S_5	48	32
S_6	73	59
S_7	34	40
S_8	59	56

The maximum mark in both examinations is the same.

You may assume that these data are a random sample from a bivariate normal distribution with mean μ_F for the French examination, mean μ_G for the German examination and Pearson's product-moment correlation coefficient ρ.

Before the examinations were sat, the Head of Languages, Pierre, decided to investigate whether there would be significant evidence of a difference between μ_F and μ_G. He decided to analyse the data using a two-tailed paired t-test with significance level 5%.

(a) Explain briefly

(i) why he chose to use a t-test and not a z-test; [1]

(ii) why he chose to use a two-tailed test and not a one-tailed test. [1]

(b) (i) State suitable hypotheses for the t-test. [1]

(ii) Find the p-value for this test. [2]

(iii) The p-value is a probability. State the event for which it gives the probability. [1]

(iv) State, giving a reason, what conclusion Pierre should reach. [2]

(This question continues on the following page)

(Question 2 continued)

(c) Pierre believes that students who score well in one language examination tend to score well in the other language examination. He therefore decides to carry out a test at the 5% significance level to investigate whether there is a positive correlation between the French examination marks and the German examination marks.

(i) State appropriate hypotheses in terms of ρ. [1]

(ii) Perform a suitable test and state the p-value. State, in context, the conclusion that Pierre should reach, giving a reason. [4]

(d) There are actually two more students in this particular class, Paul and Sue. Paul sat the French examination, but he was unable to sit the German examination. Sue sat the German examination, but she was unable to sit the French examination.

(i) Paul's mark in the French examination was 58. Use the data in **Table 1** to predict the mark that Paul would have obtained in his German examination. [3]

(ii) Based on her mark in the German examination, Sue's mark in the French examination was predicted to be 71. Find the mark she obtained in the German examination. [2]

Six students sit examinations in mathematics and history and their marks are shown in **Table 2**. The Vice Principal, Angela, decides to investigate whether there is any association between the marks obtained in these two subjects.

Table 2

Student	Mathematics mark (x)	History mark (y)
P_1	53	41
P_2	76	70
P_3	50	62
P_4	65	47
P_5	61	66
P_6	84	50

(This question continues on the following page)

(Question 2 continued)

Angela is informed that the maximum mark in each subject is 100.

Angela believes that the data might not be normally distributed, so she investigates what suitable tests are available which do not assume the data is normally distributed. She decides to use an unfamiliar test based on a statistic called Kendall's τ.

Consider n bivariate observations (x_i, y_i), $i = 1, 2, \ldots n$, such that there are no equal x-values and no equal y-values. Any pair of distinct bivariate observations (x_i, y_i) and (x_j, y_j) is said to be concordant if $(x_i - x_j)(y_i - y_j) > 0$ and discordant if $(x_i - x_j)(y_i - y_j) < 0$. For n bivariate observations, there are $\frac{n(n-1)}{2}$ distinct pairs.

Kendall's τ is defined as $\frac{2(C-D)}{n(n-1)}$ where C and D denote respectively the number of concordant pairs and discordant pairs.

(e) (i) Show that the value of Kendall's τ always lies in the interval $[-1, +1]$. [1]

(ii) For students P_1 and P_2, show that their pair is concordant. [1]

(iii) Show that the value of Kendall's τ for the mathematics and history data is 0.2. [4]

(f) Angela decided to use this statistic in a two-tailed test at the 10% significance level. The critical region for her test is $|\tau| \geq 0.733$.

(i) State, in words, her null and alternative hypotheses. [1]

(ii) State the conclusion that Angela should reach. Give a reason for your answer. [2]

Angela now finds that the history marks are actually out of 120. The history teacher advises Angela to scale the history marks so that they are out of 100 and then redo the calculations for the value of τ.

(g) State, with a reason, whether you agree with the advice given by the history teacher. [2]